Möbius transformations are a special type of rational function. Define the transformations:

\[ f(x) = \frac{ax + b}{cx + d} \]

Consider the exponent law \( a^x \cdot a^y = a^{x+y} \). This law corresponds to the property of the exponential function that it is a function of the exponent. The power function is not a function of the exponent.

Logarithms are the inverse of exponential functions. Therefore, most properties of exponentials (including duality) translate directly to those of logarithms. For example, the following four notations denote the same rational function:

\[ \log_a(x) = \log_b(x) \cdot \log_a(b) \]

Every law of exponents corresponds to a property of an exponential function. For instance, the property of the power function that \( a^x \cdot a^y = a^{x+y} \) translates to the property of the exponential function that \( e^{x+y} = e^x \cdot e^y \). Some graphs of Möbius transformations are shifted to match the axes; this is equivalent to the property that the graph is everywhere upwards sloping.

To find the equation of the horizontal asymptote, we consider the limit as \( x \to \pm\infty \). For instance, if \( c = 0 \), then the function is linear.

What is the equation of the horizontal asymptote?

\[ \lim_{x \to \pm\infty} f(x) = \pm\infty \]

In other words, if \( f(x) \) is not bounded, the horizontal asymptote is \( \pm\infty \). If \( f(x) \) is bounded, the horizontal asymptote is a horizontal line.

According to the limit as \( x \to \pm\infty \), \[ \lim_{x \to \pm\infty} f(x) = \pm\infty \]

What is the equation of the horizontal asymptote?

\[ h(x) = \pm\infty \]

The graph is upwards sloping precisely when \( x \to \pm\infty \). For example, the following four notations denote the same rational function:

\[ \frac{ax + b}{cx + d} \]

One True Hyperbola

What is the equation of the horizontal asymptote?

\[ y = \pm\infty \]

Some graphs of Möbius transformations are shifted to match the axes; this is equivalent to the property that the graph is everywhere downwards sloping.

What is the equation of the horizontal asymptote?

\[ y = \pm\infty \]

The vertical asymptote at \( x = -\frac{d}{c} \) is:

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What is the equation of the horizontal asymptote?

\[ h(x) = \pm\infty \]

The graph is downwards sloping when \( x \to \pm\infty \). For example, the following four notations denote the same rational function:

\[ \frac{ax + b}{cx + d} \]

If we take \( a \) to be a constant, then the graph is scaled by \( y \). In other words, if \( a \) is a constant, then the graph is scaled by \( y \). The graph is downwards sloping when \( x \to \pm\infty \). For example, the following four notations denote the same rational function:

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Common Graphs

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